MODELING PRESSURE FLUCTUATIONS ON A FLOW BOUNDARY

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ABSTRACT: The similarity principles of the probability characteristics of the stationary process of pressure pulsation at the boundary of an open liquid flow have been tested experimentally. The investigations were conducted in the laboratory on an ideal hydraulic jump in the range of characteristic Reynolds numbers from 4.7×10^4 to 3.7×10^5 and at the same Froude number (33) in the initial section of the jump. The scale of the flow was changed by factors of two and four. Measurements were taken at different points along the length of the jump. Pulsations whose spectrum was in the range from 0 to 50 cps were recorded. The records were processed on an electronic digital computer. It has been established that the probability characteristics of low-frequency pressure pulsations in an open flow are scaled in accordance with rules based on the laws of gravitational similarity of the phenomena. The problem of the form of the one-dimensional distribution law is briefly examined.

In solving problems connected with pressure pulsations in a liquid flow, it is important to know the rules for scaling the probability characteristics of the phenomenon from one flow scale to another.

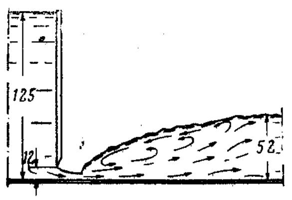

Fig. 1

The basic laws of similarity of pulsation processes in a turbulent flow can be obtained in the usual manner from the differential equations of hydrodynamics. The Euler and Strouhal numbers acquire special importance in analyzing turbulent pressure pulsations, since it is precisely these dimensionless complexes which should be employed in scaling the amplitude and frequency characteristics.

In order to study the similarity principles of pressure pulsations, systematic laboratory investigations of pressure pulsations at the bottom of an open flow in the region of a hydraulic jump were undertaken. The results of similar investigations set forth, for example, in [1-4],* are not sufficiently complete and in some respects contradictory. The behavior of the statistical characteristics of pressure pulsations with change in the geometric scale of the flow is considered below.

The experiments were conducted on an ideal hydraulic jump in a flume with a smooth horizontal bottom. The initial cross section of the jump was at a distance from the line of the flat sluice gate equal to twice the opening of the latter (Fig. 1). We shall describe the results of experiments conducted at three different geometric scales (λ = 1, 2, and 4) with the same Froude number in the initial cross section of the jump

$$F_1 = \frac{v_1^2}{gh_1} = 33.$$

The dimensions of the flow (in centimeters) to the maximum scale (λ = 1) are shown in Fig. 1. The width of the flow was not modeled, and was equal to 80 cm, that is, about 1.5 times the maximum height of the jump. The Reynolds number $R = q/\nu$ ranged from 4.7×10^4 to 3.7×10^5 (at λ = 1) (q is the specific flow rate, and ν the kinematic viscosity).

Pressure probes of the strain gauge type were installed along the axis of the flume. Their receiving areas were modeled in accordance with the geometric scale (at λ = 4, the diameter of the receiving area was 7.5 mm). The statistical characteristic of the probes was linear over the entire working range and was reproducible correct to 3% after several months of use.

Figure 2 gives the frequency characteristics of the individual components of the measuring channel: 1) probe, 2) 8ANCh-7M amplifier, and 3) electrical low-frequency filter and loop of 9SO-1 oscilliscope (East German). In this figure, f is the frequency, A/A_0 the gain, and $\varphi°$ the phase shift at frequency f. In the frequency range from 0 to 50 cps, the amplitude-frequency characteristics are almost constant and the phase-frequency characteristics are linear. Taking this into consideration and also the stationarity (in the probabilistic sense) of the processes under investigation, one may conclude that a pulsation whose spectrum is within this frequency range is recorded without important distortions. The presence of an electric filter in the measuring circuit makes it possible to exclude high-frequency noise that considerably exceeds the useful signal in this frequency range.

The pressure pulsations $p(t)$ obtained in records on the oscillograph tape were processed on an electronic digital computer in order to obtain the following statistical characteristics of the process, which was stationary:

$$\langle p \rangle = \frac{1}{T}\int_0^T p(t)\,dt, \qquad \sigma^2 = \frac{1}{T}\int_0^T [p'(t)]^2 dt, \tag{1}$$

$$r(\tau) = \frac{1}{\sigma^2 (T-\tau)}\int_0^{T-\tau} p'(t)\,p'(t+\tau)\,dt, \tag{2}$$

$$\mu_3 = \frac{1}{\sigma^3 T}\int_0^T [p'(t)]^3 dt, \quad \mu_4 = \frac{1}{\sigma^4 T}\int_0^T [p'(t)]^4 dt, \tag{3}$$

$$\Phi(x) = P\{p(t) \leqslant x\}. \tag{4}$$

Here $\langle p \rangle$ is the averaged pressure; σ the mean square value of pressure pulsations; $r(t)$ the normalized autocorrelation function; μ_3 and μ_4 are the coefficients of skewness and kurtosis respectively; $\Phi(x)$ is the operator for finding the statistical probability (frequency) of the event indicated in parentheses; t, T, τ represent time; $p'(t) = p(t) - \langle p \rangle$.

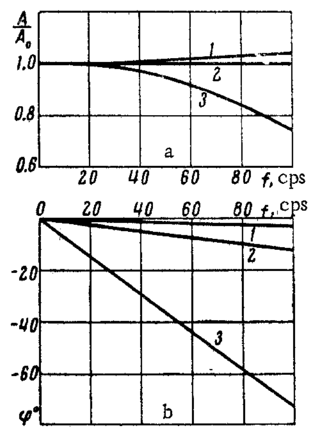

Fig. 2

*Reviews of other studies of this problem are given in [2, 3].

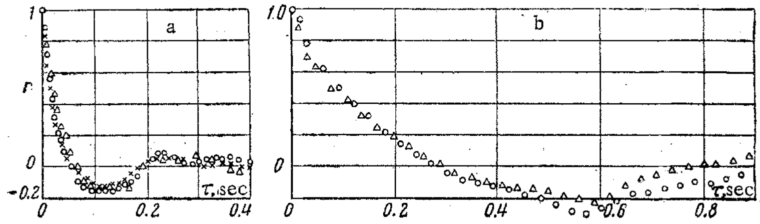

Fig. 3

The averaging period T was taken (as a result of check computations) to be no less than:* 50 sec for $\lambda = 1$, 36 sec for $\lambda = 2$, and 25 sec for $\lambda = 4$. Such a high value (about 100 periods of the lowest pulsation frequency) was connected with the nonstationarity of the position of the jump relative to the probe. The large value of T is also explained by the insensitivity, noted during the check computations, of these statistical characteristics to changes in the quantization step of the processes in time Δt from 0.008 to 0.04 sec (from 10 to 2 points for the smallest pulsation period). This made it possible to lay out the ordinates with the same step $\Delta t = 0.008$ sec at all three scales λ.

As is known from analyses of the Navier-Stokes equations, all all the following numbers should in general be equal in nature and in the model in similar hydrodynamic phenomena: Froude F, Reynolds R, Euler E, and Strouhal S (the principal numbers for the majority of cases):

$$F = \frac{v_0^2}{gl_0}, \quad R = \frac{v_0 l_0}{\nu}, \quad E = \frac{\delta p_0}{\rho v_0}, \quad S = \frac{l_0}{v_0 t_0}.$$

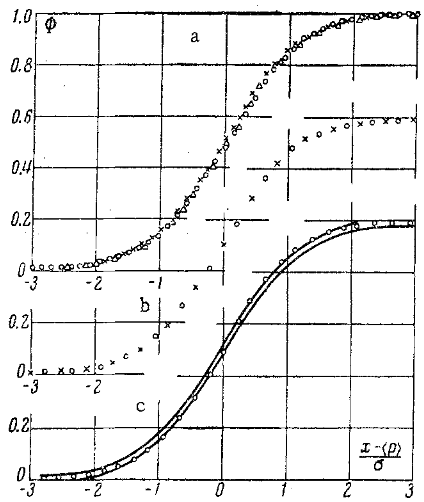

Fig. 4

Here v_0, l_0, t_0, δp_0 are the characteristic values of the velocity, length, time, and pressure, respectively; g is the acceleration of gravity, ν is the viscosity, and ρ the density. In hydrodynamic phenomena where inertial and gravitational forces play the predominant parts, even at comparatively small Reynolds numbers, self-similarity is achieved with respect to the Reynolds number and the only characteristic criterion for the averaged flow parameters is the Froude number.

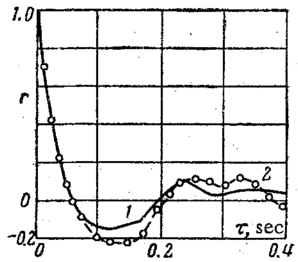

Fig. 5

In the case of the probability characteristics of a pulsation process, however, the assumption of self-similarity with respect to Reynolds numbers and the decisive role of the Froude number (Froude modeling) requires experimental confirmation. Here we have in mind that it is necessary to make use of the Euler and Strouhal numbers in order to scale the amplitude and frequency characteristics of pressure pulsations [5].

From the practical standpoint, the low-frequency part of the pressure pulsation spectrum is the most interesting. According to modern ideas of turbulence theory, the large-scale vortices "responsible" for these frequencies are completely defined by averaged motion. Thus, the above-mentioned assumptions concerning this part of the pressure pulsation spectrum are fully justified. As for small-scale turbulence, we can expect more complicated modeling laws.

Now, on the basis of previously obtained experimental data, we shall test the relationships which follow from the modeling principles set forth above (the subscript 1 corresponds to $\lambda = 1$, while the subscript λ corresponds to the scale $\lambda \neq 1$)

$$\langle p_1 \rangle = \lambda \langle p_\lambda \rangle, \quad \sigma_1 = \lambda \sigma_\lambda, \tag{5}$$

$$r_1(\tau) = r_\lambda(\tau / \sqrt{\lambda}). \tag{6}$$

$$(\mu_3)_1 = (\mu_3)_\lambda, \quad (\mu_4)_1 = (\mu_4)_\lambda, \quad \Phi_1\left(\frac{x - \langle p_1 \rangle}{\sigma_1}\right) = \Phi_\lambda\left(\frac{x - \langle p_\lambda \rangle}{\sigma_\lambda}\right). \tag{7}$$

The table illustrates the behavior of the numerical characteristics of the pressure pulsations with change in the geometric scale λ ($\langle p \rangle$ and σ are given in millimeters of water column). The quantity $\eta = l/l_n$ denotes the distance from the initial cross section of the jump in fractions of the length of the horizontal projection of the cylinder

*This relationship of minimum averaging periods in different linear scales λ corresponds to the law of time scaling in modeling phenomena according to the laws of gravitational similarity.

	$\eta = 0.4$			$\eta = 0.6$			$\eta = 0.8$		$\eta = 1.0$	
	$\lambda = 1$	$\lambda = 2$	$\lambda = 4$	$\lambda = 1$	$\lambda = 2$	$\lambda = 4$	$\lambda = 1$	$\lambda = 4$	$\lambda = 1$	$\lambda = 4$
$\langle p \rangle$	346	171	86	338	163	80	474	—	500	—
σ	56.0	27.9	13.0	36.5	19.0	8.3	22.8	6.1	16.0	3.8
μ_3	−0.47	0.14	−0.05	−0.25	−0.14	−0.40	0.03	−0.06	0.00	0.25
$\mu_4 - 3$	1.06	2.45	1.45	0.77	0.90	1.35	0.26	0.74	1.79	0.39

l_n computed by means of M. D. Chertousov's formula [6].* The maximum deviation obtained in the experiments was equal to

$$\Delta\sigma = \frac{\sigma_1 - \lambda\sigma_\lambda}{\sigma_1} 100\% = 7\% \cdot$$

This is evidence that the mean square value can be modeled by Froude modeling. The coefficients of skewness and kurtosis are most sensitive to experimental and processing errors owing to their smallness. Their behavior pattern with change in scale was not observed, thus the data in the table do not refute (7).

Normalized autocorrelation functions scaled according to (6) for $\lambda = 1$ are presented in Fig. 3 as an illustration, Figs. 3a and 3b corresponding to values of $\eta = 0.4$ and 1.0. In this and the following figures, the points correspond to $\lambda = 1$, the crosses to $\lambda = 2$, and the triangles to $\lambda = 4$. These functions agree satisfactorily along the entire length of the jump.

The one-dimensional integral laws of distribution of centered and normalized ordinates of processes written in different scales and at points with coordinates $\eta = 0.4$ and 1.0 are plotted in Figs. 4a and 4b. A check showed that all the points lie well within the confidence regions [7] whose boundaries correspond to 10% (Fig. 4a) and even 50% (Fig. 4b) fiducial probability. Two conclusions follow from this. In the first place, there are no grounds for refuting the hypothesis that different samples belonging to the same general population, that is, the coincidence of centered and normalized distribution laws on models of equal scale. In the second place, errors in determining the distribution laws connected with the finiteness of the averaging period T turned out to be insignificant (in this connection, the conclusions concerning Fig. 4c are more convincing). Similar results also hold for other points along the length of the jump. Thus, the above-mentioned probability characteristics in the low-frequency part of the pressure pulsation spectrum can be modeled by Froude modeling in all zones of the hydraulic jump.

In conclusion, we shall consider the problem of the form of the distribution law. In Fig. 4c, the solid lines define the confidence region of the normal distribution law in accordance with A. N. Kolmogorov's criterion. In this case, the fiducial probability is taken to be equal to 5%. The points correspond to the experimental distribution law obtained by averaging over all scales with $\eta = 0.4$. The fact that a part of the experimental points lie outside the boundaries of the fairly wide confidence region is evidence that the distributions obtained in the experiments differ from the normal distribution. The substantial values of the excesses given in the table lead to the same conclusion.

However, when applying the statistical method, the difference between the processes considered and normal processes may be neglected in some cases. For example, Fig 5 illustrates satisfactory agreement of normalized autocorrelation functions calculated 1) from the general algorithm (2), and 2) from the algorithm $r(\tau) = \cos\pi\eta$, which is valid only for normal processes. Here η is the probability that the ordinates of a centered process taken with step τ are of the same sign.

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*The length of the jump considered here serves only to determine the coordinates of the measuring points, thus the selection of this or that empirical formula (satisfying the requirements of similarity theory) is not of fundamental importance. For the same Froude number, the corresponding linear dimensions vary according to a linear scale.

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